

Navigating the Lobbying Landscape: Insights from Opinion Dynamics Models

Daniele Giachini^{1,*}, Leonardo Ciambezi¹, Verdiana Del Rosso², Fabrizio Fornari²,
Valentina Pansanella³, Lilit Popoyan^{4,1}, and Alina Sîrbu⁵

¹Sant'Anna School of Advanced Studies, Institute of Economics, Pisa (PI), 56127, Italy

²University of Camerino, Department of Computer Science, Camerino (MC), 62032, Italy

³National Research Council (CNR), Institute of Information Science and Technologies "A. Faedo" (ISTI), Pisa (PI), 56124, Italy

⁴Queen Mary University of London, School of Business and Management, London, E14NS, United Kingdom

⁵University of Bologna, Department of Computer Science and Engineering, Bologna (BO), 40126, Italy

*daniele.giachini@santannapisa.it

ABSTRACT

While lobbying has been demonstrated to have an important effect on public opinion and policy making, existing models of opinion formation do not specifically include its effect. In this work we introduce a new model of opinion dynamics where lobbyists can implement complex strategies and are characterised by a finite budget. Individuals update their opinions through a learning process resembling Bayesian learning, but influenced by cognitive biases such as under-reaction and confirmation bias. We study the model numerically and demonstrate rich dynamics both with and without lobbyists. In the presence of lobbying, we observe two regimes: one in which lobbyists can have full influence on the agent network, and another where the peer-effect generates polarisation. When symmetric lobbyists are present, the lobbyist influence regime is characterised by long opinion oscillations, while in the transition area between the two regimes we observe convergence to the optimistic model when the lobbying influence is long enough. These rich dynamics pave the way for studying real lobbying strategies to validate the model in practice.

Introduction

Lobbying is a pervasive feature of modern policymaking, shaping decisions across virtually all domains of public life, from environmental regulation and healthcare to financial oversight and digital governance. Beyond direct engagement with policymakers, lobbying strategies increasingly aim to influence public opinion, leveraging social networks and mass communication to create political momentum, reduce resistance to controversial policies, or shift the Overton window on key issues. Lobbyists, whether representing corporate interests, civil society, or ideological coalitions, act strategically to influence not only policymakers, but also public perception and discourse. A large body of empirical literature has demonstrated the effectiveness of lobbying in shaping policy outcomes^{1,2}, with recent work underscoring the growing sophistication of lobbying strategies in networked political and media environments^{3,4}. This broader landscape of influence highlights the need to understand lobbying not only as a transactional activity, but also as a dynamic process of belief formation and diffusion in complex social systems.

Among the various mechanisms through which lobbying exerts influence, opinion formation within the public sphere plays a critical role. Lobbyists increasingly target public attitudes to create political pressure or legitimacy for specific policy choices^{5,6}. The advent of digital platforms has magnified these efforts, enabling tailored messaging and decentralized opinion shaping. This makes the diffusion of opinions across social networks, where individuals interact, update beliefs, and potentially become advocates themselves, a crucial site for understanding lobbying efficacy.

To study this complex landscape, opinion dynamics models offer powerful tools. Opinion dynamics models are mathematical representations utilized to elucidate the intricate mechanisms underlying the dissemination and evolution of opinions or beliefs across heterogeneous populations^{7,8}. These models, rooted in theories of social influence and information diffusion, and also inspired by models from statistical physics, have garnered substantial attention and application across diverse disciplines, including sociology⁹, political science¹⁰, marketing¹¹ and economics¹². They offer valuable insights into the emergence of collective behaviors, the formation of consensus, and the dynamics of information propagation within social networks¹³⁻¹⁵. Recently, they have also been used to model opinion formation in settings mediated by technology, for example through a bounded confidence model with algorithmic bias^{16,17}, which demonstrated how interaction through online social media that

promotes information exchange with similar peers can cause fragmentation and polarization of opinions. In economic modelling and political science, opinion dynamics models have increasingly intersected with agent-based modeling approaches^{18–20}, allowing for the incorporation of bounded rationality, motivated reasoning, and strategic behavior.

Despite their extensive utilization in numerous domains, the application of opinion dynamics models within the lobbying realm remains a nascent and underexplored area of inquiry. While existing research has shed light on the role of social networks and public discourse in shaping climate attitudes and policy preferences²¹, the integration of formalized mathematical frameworks to analyze and forecast lobbying dynamics represents a promising frontier for advancing our understanding of the complex interplay between public opinion, lobbying discourse, and policy outcomes. However, relatively few models explicitly integrate lobbying agents into these frameworks. Many studies consider mass media influence in opinion dynamics, typically modelled as external field that can influence all agents with a certain probability^{13–15, 17, 22}. The effects observed depend on the structure of the population and model parameters: while in some situations mass media can stimulate consensus or speedup convergence, in other cases, especially when promoting extreme messages, it can facilitate polarisation. Alternatively, models can include zealots or stubborn agents^{15, 23}, with similar effects. While some similarities exist, these approaches do not fully encompass the concept of lobbying, which can implement different dynamic strategies and typically have a cost. Furthermore, the modelling of opinions in these examples is very simplified, sometimes including bounded confidence or disagreement, but not taking into account more complex mechanisms such as confirmation bias or motivated reasoning. Therefore, the endogenous modeling of lobbying strategies aimed at belief manipulation within a dynamic opinion formation process remains underdeveloped.

This paper contributes to this growing interdisciplinary literature by introducing a novel model of lobbying influence over opinion dynamics in a boundedly rational social network. In our framework, a population of agents interacts repeatedly to update their subjective beliefs about the likelihood of an uncertain future event, for example, a damaging climate event. Each agent weighs two competing probabilistic models—one “optimistic” and one “pessimistic”—and updates their beliefs through Bayesian-like learning, modulated by under-reaction^{24–26} and confirmation bias (a form of directional motivated reasoning)²⁷. Agents are embedded in a directed network and interact through sequential signal exchanges that propagate beliefs over time. We then endogenize lobbying by introducing external agents — lobbyists — who strategically send costly signals to influence the population’s belief distribution by the end of a finite time horizon. Lobbyists differ in the model they support and are subject to a resource constraint (budget). Their objective is to minimize the distance between the final average belief and their preferred model. Lobbyists select randomized strategies from a constrained set of signaling paths, and their interventions shape both the speed and direction of belief evolution in the network.

Our numerical results reveal rich dynamics. In the absence of lobbying, the network tends to converge toward consensus or, under high confirmation bias, become polarized. A single lobbyist with sufficient resources can dominate belief formation and steer the network toward its preferred model, especially when agents are relatively open to new information. However, when two lobbyists with opposing goals act simultaneously, the system often fails to converge within the active time horizon, resulting in persistent instability and oscillation — especially in regions of the parameter space where both lobbyists are highly effective. We also uncover non-trivial path dependence and sensitivity to initial conditions, pointing to the complex interplay between cognitive constraints, network structure, and strategic influence.

The remainder of the paper is structured as follows. The next section introduces the baseline opinion dynamics model, describing the agent interaction protocol and belief updating mechanism, and then incorporates lobbying agents and formalizes their strategic behavior, cost constraints, and impact on individual beliefs. The Results section presents the simulation results across three scenarios: no lobbying, single-lobbyist, and dual-lobbyist configurations. It explores the model’s behavior across key parameters (underreaction and confirmation bias), and performs sensitivity analysis on budget, network size and time horizon. We then conclude by summarizing the findings and suggesting avenues for future research.

Methods: model description

The baseline model

Consider N individuals who interact for τ rounds about the occurrence of an uncertain damaging event after the interaction rounds. Individuals do not know the true probabilities that determine the occurrence of the event, but can rely on two probabilistic models. Those are the probability distributions $(\pi_o, 1 - \pi_o)$ and $(\pi_p, 1 - \pi_p)$, where π_k , with $k \in \{o, p\}$, indicates the probability that the damaging event occurs. The model $(\pi_o, 1 - \pi_o)$ is *optimistic* while the model $(\pi_p, 1 - \pi_p)$ is *pessimistic*: $0 < \pi_o < \pi_p < 1$ with π_o close to zero and π_p close to one.

The social network is represented by a *directed* graph, where a link from i to j means that agent i communicates to agent j but not viceversa (for example followers in online social networks). We represent this by the $N \times N$ adjacency matrix $A = (a_{i,j})$. Communication occurs in the interaction rounds. In each round $t \in \{1, \dots, \tau\}$, an individual t_i is independently and uniformly drawn from the population and it sends a signal $s_t \in \{0, 1\}$ to the set of individuals to whom it communicates, that is

$J(t) = \{j \in \{1, 2, \dots, N\} | a_{t,j} = 1\}$. The signal $s_t = 1$ indicates that the individual communicates that the event will occur, while $s_t = 0$ indicates that the individual communicates that the event will not occur.

Each individual i has a subjective probability distribution $\mathbf{p}_{i,t} = (p_{i,t}, 1 - p_{i,t})$ on the realization of the event, with $p_{i,t}$ the probability attached at round t to the case in which the event occurs. Such a distribution evolves according to the signals the individual receives from its network of connections and it is used to draw the signals it sends to the individuals to whom it communicates. Concerning the evolution of subjective probabilities, for any $i \in \{1, 2, \dots, N\}$ and $t \in \{1, \dots, \tau\}$, it is

$$p_{i,t} = w_{i,t} \pi_o + (1 - w_{i,t}) \pi_p \quad (1)$$

with $w_{i,t} \in (0, 1)$. That is, any individual builds its subjective probabilities as the convex combination of the probabilistic predictions of the two models. The weights $w_{i,t}$ used to build subjective probabilities change depending on the received signals. In particular, for any individual i and for any round t , the weight assigned to the optimistic model evolves according to

$$w_{i,t} = \begin{cases} w_{i,t-1} & \text{if } i \notin J(t), \\ \lambda_{i,t} w_{i,t-1} + (1 - \lambda_{i,t}) w_{i,t-1} \left(s_t \frac{\pi_o}{p_{i,t-1}} + (1 - s_t) \frac{1 - \pi_o}{1 - p_{i,t-1}} \right) & \text{if } i \in J(t), \end{cases} \quad (2)$$

where $\lambda_{i,t} \in [0, 1]$ modulates the effects of cognitive biases of agent i at round t and $w_{i,0} \in (0, 1)$ is the initial prior. If $\lambda_{i,t} = 0$ then i is a Bayesian learner and updates probabilities according to Bayes rule²⁸. If $\lambda_{i,t} = 1$ then i keeps the initial combination fixed no matter the sequence of signals it receives. Intermediate values of $\lambda_{i,t}$ indicate that, as i receives a signal, it updates the weights in the right direction but in lower magnitude than what Bayesian learning would prescribe. Thus, individuals under-react to information^{24–26} and we enrich such a behavioural interpretation imposing that $\lambda_{i,t}$ depends upon the type of signal received and the prior. In this way, it captures a form of directional motivated reasoning that, in turn, gives rise to confirmation bias (the tendency of individuals to discard information that contradicts their priors), a common feature in the formation of beliefs²⁷. In particular, we assume

$$\lambda_{i,t} = \phi_i |1 - s_t - w_{i,t-1}| + (1 - \phi_i) \lambda_i \quad (3)$$

with $\phi_i, \lambda_i \in [0, 1]$. That is, the overall degree of under-reaction is the convex combination of two components. On the one hand, the parameter λ_i captures the baseline level of under-reaction of agent i . On the other hand, the function $|1 - s_t - w_{i,t-1}|$ introduces directional motivated reasoning or, equivalently for our analysis and interpretation, confirmation bias. The parameter ϕ_i regulates the strength of the directional motivated reasoning. Suppose, for instance, that directional motivated reasoning is strong ($\phi_i \simeq 1$), if an individual has a strong prior in the optimistic model ($w_{i,t-1} \simeq 1$) and receives a signal that favours it with respect to the pessimistic model ($s_t = 0$) then its overall degree of under-reaction is low: information has a sensible impact on its beliefs. If, instead, the information contradicts its priors ($s_t = 1$), then the degree of under-reaction is high: information has a negligible effect on its beliefs. A symmetric argument holds if the agent's priors favour the pessimistic model. If, instead, directional motivated reasoning is weak ($\phi_i \simeq 0$), then the individual under-reacts with respect to information because of the effect of λ_i , but the nature of the signal and its relation to the prior have a negligible effect on probability updating.

Assuming that the signal at round t is drawn from the distribution $\mathbf{p}_{t,t}$ (i.e., $\text{Prob}\{s_t = 1\} = p_{t,t}$), the timeline of events in each round t is as follows: *i*) t_t is independently and uniformly drawn from the population; *ii*) the set of receivers $J(t)$ is created; *iii*) the signal s_t is drawn from the distribution $\mathbf{p}_{t,t}$; *iv*) individual weights are updated according to Eq. (2).

Introducing lobbyists

Lobbyists can be considered external agents with respect to the social network that, in each turn, can pay a cost to send signals to individuals. Their objective is to influence the time τ distribution of beliefs so that it becomes concentrated on one of the two models.

Assume that there are L active lobbyists and define $S_{i,t}^\ell \in \{0, 1\}$ as a variable indicating whether lobbyist $\ell \in \{1, 2, \dots, L\}$ sends a signal to individual i at the beginning of round t ($S_{i,t}^\ell = 1$) or not ($S_{i,t}^\ell = 0$). It follows that a pure strategy for lobbyist ℓ can be indicated as a matrix $S^\ell \in \mathcal{S} = \{0, 1\}^{N \times \tau}$. Each lobbyist supports one model between the optimist and the pessimist one and the signals it sends are in favour of the model it supports. For instance, a lobbyist supporting the optimistic model will signal 0, that is, the event is not occurring. Define the function that assigns to each lobbyist the model it supports as $m : \{1, 2, \dots, L\} \rightarrow \{o, p\}, \ell \mapsto m(\ell)$. We assume that signals are costly. In particular, the cost of a signal is fixed to a given level $c > 0$ and each lobbyist ℓ has a budget B^ℓ . Without loss of generality, we shall set $c = 1$ in what follows. That is, B^ℓ shall be understood as the maximum number of signals lobbyist ℓ can send. Notice that this assumption constrains the set of strategies that each lobbyist can actually play, that is, a feasible strategy is such that $\sum_{t=1}^{\tau} \sum_{i=1}^N S_{i,t}^\ell \leq B^\ell$. Hence, we define the

set of feasible strategies for lobbyist ℓ as $\mathcal{S}_\ell = \{S^\ell \in \mathcal{S} \mid \sum_{t=1}^T \sum_{i=1}^N S_{i,t}^\ell \leq B^\ell\}$. We allow lobbyists to randomize their choices. Call $\Delta(\mathcal{S}_\ell)$ the mixed extension of $\mathcal{S}_\ell$ and $\sigma^\ell(S)$ the probability that lobbyist ℓ attaches to a feasible strategy S , such that $\sigma^\ell \in \Delta(\mathcal{S}_\ell)$ is a feasible mixed strategy for lobbyist ℓ .

Each individual reacts to the signal received in the same way as they do for peer signals. Therefore, now, individual probabilities also evolve as a consequence of the strategies of the lobbyists. In particular, call $S^{-\ell} = (S^1, \dots, S^{\ell-1}, S^{\ell+1}, \dots, S^L) \in \mathcal{S}_1 \times \dots \times \mathcal{S}_{\ell-1} \times \mathcal{S}_{\ell+1} \times \dots \times \mathcal{S}_L$ a profile of feasible pure strategies for the lobbyists different from ℓ , such that $p_{i,\tau}(S^\ell, S^{-\ell})$ indicates the final probability of individual i as a function of the strategies played by all lobbyists. Define the vector of lobbyists' signals received by agent i at time t as $\mathcal{Z}^{i,t} \in \{0, 1, 2\}^L$, where $\mathcal{Z}_\ell^{i,t} = 0$ if $S_{i,t}^\ell = 0$, $\mathcal{Z}_\ell^{i,t} = 1$ if $S_{i,t}^\ell = 1$ and $m(\ell) = p$, $\mathcal{Z}_\ell^{i,t} = 2$ if $S_{i,t}^\ell = 1$ and $m(\ell) = o$. Thus, assume that lobbyists send signals at the beginning of each round t and that the order in which their signals are received by individuals is $Z_t \in \text{Perm}\{1, \dots, L\}$, i.e. an element of the set of all permutations of the indexes of lobbyists (in what follows, we assume that the order according to which lobbyists send their signals is randomly chosen). The weight assigned to the optimistic model deriving from previous interaction needs to be updated for any $z \in \{1, \dots, L\}$ according to

$$w_{i,t-1}^z = \begin{cases} w_{i,t-1}^{z-1} & \text{if } \mathcal{Z}_{\ell_z}^{i,t} = 0, \\ \lambda_{i,t}^{1,z} w_{i,t-1}^{z-1} + (1 - \lambda_{i,t}^{1,z}) w_{i,t-1}^{z-1} \frac{\pi_o}{\pi_o w_{i,t-1}^{z-1} + \pi_p (1 - w_{i,t-1}^{z-1})} & \text{if } \mathcal{Z}_{\ell_z}^{i,t} = 1, \\ \lambda_{i,t}^{2,z} w_{i,t-1}^{z-1} + (1 - \lambda_{i,t}^{2,z}) w_{i,t-1}^{z-1} \frac{1 - \pi_o}{1 - \pi_o w_{i,t-1}^{z-1} - \pi_p (1 - w_{i,t-1}^{z-1})} & \text{if } \mathcal{Z}_{\ell_z}^{i,t} = 2, \end{cases} \quad (4)$$

where ℓ_z indicates the lobbyist index number appearing in z -th position of the Z_t vector, $w_{i,t-1}^0 = w_{i,t-1}$, $\lambda_{i,t}^{1,z} = \phi_i w_{i,t-1}^{z-1} + (1 - \phi_i) \lambda_i$, and $\lambda_{i,t}^{2,z} = \phi_i (1 - w_{i,t-1}^{z-1}) + (1 - \phi_i) \lambda_i$. In this way, the cognitive biases discussed in advance influence belief updating also when signals are sent by lobbyists. The weight updating rule for the signals received from other individuals needs to be adjusted, indeed, equation (2) now becomes

$$w_{i,t} = \begin{cases} w_{i,t-1}^L & \text{if } i \notin J(t), \\ \lambda_{i,t} w_{i,t-1}^L + (1 - \lambda_{i,t}) w_{i,t-1}^L \left(s_t \frac{\pi_o}{\pi_o w_{i,t-1}^L + \pi_p (1 - w_{i,t-1}^L)} + (1 - s_t) \frac{1 - \pi_o}{1 - \pi_o w_{i,t-1}^L - \pi_p (1 - w_{i,t-1}^L)} \right) & \text{if } i \in J(t), \end{cases} \quad (5)$$

with $\lambda_{i,t} = \phi_i |1 - s_t - w_{i,t-1}^L| + (1 - \phi_i) \lambda_i$ as in advance. Subjective probabilities, instead, remain as in equation (1) since they are computed at the end of the interaction round.

Finally, we assume that the payoff of lobbyist ℓ using the feasible mixed strategy σ^ℓ when the other lobbyists use the feasible mixed strategies $\sigma^1, \dots, \sigma^{\ell-1}, \sigma^{\ell+1}, \dots, \sigma^L$ is

$$U_\ell = -\mathbb{E} \left[\sum_{S^1 \in \mathcal{S}_1} \dots \sum_{S^L \in \mathcal{S}_L} \prod_{l=1}^L \sigma^l(S^l) \left| \frac{1}{N} \sum_{i=1}^N p_{i,T}(S^\ell, S^{-\ell}) - \pi_{m(\ell)} \right| \right],$$

where the expectation $\mathbb{E}$ is computed with respect to the random variables deciding the order in which lobbyists send their signals, the selection of communicating individuals, and the signalling choices operated by individuals. The underlying intuition is that a lobbyist benefits as the expected distance between the average final belief and the model it supports decreases.

When lobbyists are active, the timeline of events in each round t becomes: *i*) each lobbyist sends the signals to the individuals it selected; *ii*) individual weights are updated according to equation (4); *iii*) t_t is independently and uniformly drawn from the population; *iv*) the set of receivers $J(t_t)$ is created; *v*) the signal s_t is drawn from the distribution $\mathbf{p}_{t,t}$; *vi*) individual weights are updated according to equation (5).

Example 1: one individual and one rational lobbyist

A simple illustrative example is the special case in which $N = L = 1$. Assume that the lobbyist supports the pessimistic model and that $w_{i,0}, \lambda_1, \phi_1 \in (0, 1)$. By direct inspection of equation (4), one notices that the weight assigned to the optimistic model decreases if $S_{1,t}^1 = 1$ (for any t). This implies that the payoff of the lobbyist increases for each signal sent. Thus, a lobbyist that wants to maximize its payoff will use all of its budget to send signals. Since the specific timing of the signals does not matter, the optimal strategy of the lobbyist is

$$S^{1,*} = \begin{cases} (1, \dots, 1) & \text{if } \tau \leq B^1, \\ S^1 \in \{(S_{1,1}^1, \dots, S_{1,\tau}^1) \mid \sum_{t=1}^T S_{1,t}^1 = \lfloor B^1 \rfloor\} & \text{if } \tau > B^1. \end{cases}$$

Example 2: two unbiased individuals, one period, and two rational lobbyists with unitary budget supporting opposite models

The second special case we consider is characterised by $N = L = 2$, $\tau = 1$, $B^1 = B^2 = 1$, and $\lambda_i = \phi_i = 0 \forall i \in \{1, 2\}$. Without loss of generality, we assume that lobbyist 1 supports the pessimistic model while lobbyist 2 supports the optimistic model. Since both time and budget are unitary (and not sending signals is strictly dominated), each lobbyist has two feasible pure strategies: sending a signal to individual 1 or to individual 2, that is, $S^\ell \in \mathcal{S}_\ell = \{(0, 1), (1, 0)\}$. Under the assumptions that the model structure is common knowledge and the lobbyists are rational, we can compute the Nash equilibrium²⁹ resulting from the strategic interaction of the two lobbyists.

Notice that $\lambda_i = \phi_i = 0 \forall i \in \{1, 2\}$ implies that both individuals update their beliefs according to Bayes rule, hence, the order in which they receive signals does not matter. Thus, to compute final probabilities, we only need to consider the following set of possible received signals: $\Omega = \{\emptyset, \{0\}, \{1\}, \{0, 0\}, \{0, 1\}, \{1, 1\}, \{0, 0, 1\}, \{0, 1, 1\}\}$. That is, since the lobbyists support opposite models and one of the individuals sends one signal to the other, the most extreme cases are the one in which an individual does not receive any signal, and the one in which it receives two (different) signals from the lobbyists and a signal from the other agent. All the other cases in between are possible. Assuming a uniform prior distribution for both individuals, the two agents are *ex-ante* identical, meaning that the differences in their final probabilities are uniquely generated by the signals they received. Hence, we can simplify the notation calling $w(\omega)$ the weight assigned to the optimistic model by an agent that has received the signals $\omega \in \Omega$. As a direct consequence, we define the probability such an agent assigns to the realization of the event as $p(\omega) = w(\omega)\pi_o + (1 - w(\omega))\pi_p$. Then, calling t_s , with $s \in \{0, 1\}$, the number of times s occurs in ω , one has

$$w(\omega) = \frac{\pi_o^{t_1}(1 - \pi_o)^{t_0}}{\pi_o^{t_1}(1 - \pi_o)^{t_0} + \pi_p^{t_1}(1 - \pi_p)^{t_0}}.$$

Given the previous assumptions, the symmetry they entail, and the fact that $p(\omega) \in (\pi_o, \pi_p) \forall \omega$, the payoff of lobbyist $\ell \in \{1, 2\}$ conditional upon both lobbyists playing the same pure strategy ($S^1 = S^2$, lobbyists send their signals to the same individual) is

$$u_\ell(S^1 = S^2) = - \left| \frac{1}{4} (p(0, 1)(1 + p(1)) + (1 - p(0, 1))p(0) + p(\emptyset)(1 + p(0, 1, 1)) + (1 - p(\emptyset))p(0, 0, 1)) - \pi_{m(\ell)} \right|,$$

while the payoff of ℓ conditional upon the two lobbyists choosing different pure strategies ($S^1 \neq S^2$, lobbyists send their signals to different individuals) is

$$u_\ell(S^1 \neq S^2) = - \left| \frac{1}{4} (p(1)(1 + p(0, 1)) + (1 - p(1))p(0, 0) + p(0)(1 + p(1, 1)) + (1 - p(0))p(0, 1)) - \pi_{m(\ell)} \right|.$$

Thus, for a generic mixed strategy profile, the payoff of lobbyist ℓ reads

$$U_\ell = (\sigma^1(1, 0)\sigma^2(1, 0) + \sigma^1(0, 1)\sigma^2(0, 1))u_\ell(S^1 = S^2) + (\sigma^1(0, 1)\sigma^2(1, 0) + \sigma^1(1, 0)\sigma^2(0, 1))u_\ell(S^1 \neq S^2).$$

However, notice that $u_1(S^1 = S^2) - u_1(S^1 \neq S^2) = u_2(S^1 \neq S^2) - u_2(S^1 = S^2)$, hence, neglecting the trivial case in which the differences are zero, there generically exists a unique Nash equilibrium characterised by $\sigma^\ell(1, 0) = \sigma^\ell(0, 1) = 1/2 \forall \ell \in \{1, 2\}$.

Simulation setup

In all of our simulations, we consider a complete network of 500 agents. The adjacency matrix A is therefore of size 500×500 , with diagonal entries $a_{i,i} = 0$, and off-diagonal entries $a_{i,j \neq i} = 1$. Each agent is characterised by an individual-specific initial weight $w_{i,0}$ which is randomly drawn from a uniform distribution with support $(0, 1)$. The probabilities characterizing the two models (optimistic and pessimistic) are $\pi_o = 0.01$ and $\pi_p = 0.99$. We consider three main scenarios. In the “baseline” scenario, the network is left to interact with itself, with no outside interference, i.e. no lobbyist. In the “one-lobbyist” scenario, we introduce a single active lobbyist ($L = 1$) which supports the pessimistic model ($m(1) = p$). In the “two-lobbyists scenario”, we introduce two active lobbyists that support competing models ($m(1) = p$, and $m(2) = o$). In each scenario contemplating lobbyists, each one of them is endowed with a budget $B = 10,000$. For the purpose of our numerical analysis, we introduce the parameter $T \leq \tau$, representing the lobbyists’ time horizon, that is, the time span over which it allocates its budget B^ℓ . Accordingly, we set $S_{i,t}^\ell = 0 \forall t \in (T, \tau]$. This constraint allows us to manage the potentially vast space of feasible lobbying strategies and enables analysis in scenarios with large τ . In particular, neglecting the zeros appearing after T , we assume that lobbyists dispose of a set of 100 strategy matrices of size $(T \times 500)$ in which the variable indicators for signals are distributed randomly over nodes and iterations, and choose one of those matrices at random at the beginning of each independent simulation run. For most of our experiments, we set $T = 100$. This means that, over 100 periods, each lobbyist is able to send 100 signals per period, thus reaching out to 20% of the nodes at each time-step. Moreover, this strategy selection procedure resembles an adaptation of the uniform-probability mixed strategy that emerges as the Nash equilibrium in Example 2.

As shown in equation (3), each $\lambda_{i,t}$ is subject to two different types of bias, captured by the parameters λ_i and ϕ_i . While λ_i captures the degree of under-reaction to *any* new information, ϕ_i represents the strength of directional motivated reasoning (or confirmation bias), which is signal-dependent. We explore the properties of the model assuming homogeneous biases ($\lambda_i = \lambda$ and $\phi_i = \phi$ for all individuals i) and considering different configurations of the network with respect to these two independent dimensions, i.e. a parameter space generated by $(\lambda, \phi) \in [0, 1] \times [0, 1]$.

In all simulations, we monitor various criteria. An important aspect is the number of clusters that form in the population: consensus means one cluster, more clusters mean fragmentation and polarisation. We employ the effective number of clusters defined as $C = N^2 / \sum_i N_i^2$, where k is the number of clusters and N_i is the size of each cluster. This measure takes into account not only the number of groups, but also their size. C is equal to k when we have k clusters of equal size, and it is smaller when the clusters are imbalanced. A second criterion of interest is the average opinion of agents at the end of the simulation, which we report in terms of average of the $p_{i,\tau}$ values as defined in Eq. 1. In addition to indicating how beliefs have collectively shifted, this criterion is also informative of lobbyists' payoff. Furthermore, we look at the number of iterations required for convergence, indicating the speed of opinion cluster formation. For all parameter configurations we provide mean values of these criteria over 150 independent runs, each one of which is simulated until the model reaches convergence (i.e., until all the individual weights assigned to the competing models stop updating).

Results

Baseline scenario

In the first setting, we investigated the properties of the model in the absence of any active lobbyist, when the nodes are left free to interact with their peers without outside input. Figure 1 displays the effective number of clusters and the average subjective probabilities across the parameter space when the stable state is reached. We observe that, for most of the parameter region under consideration, the distribution of subjective probabilities tends to converge to a general consensus, with all of the nodes eventually supporting a single model (either o or p , depending on initial conditions). This dynamic is relatively quick and is also illustrated by the evolution plot in Fig. 2a.

Two notable exceptions to this behavior are worth mentioning: the first, and more trivial, occurs in the special case where $\lambda = 1$ and $\phi = 0$. In this case, as it is immediately clear from equation (3), nodes are “deaf” to whatever signal they receive from their peers, and any update of subjective probabilities is impossible. In this case, there is no dynamics and each agent maintains its initial condition. The second exception emerges at the bottom and to the right of the parametric region, where the confirmation bias parameter ϕ is sufficiently large to prevent the network from reaching a consensus. In this case, in many runs the model converges to a polarized equilibrium, in which a variable proportion of agents supports one model, while the rest of the network supports the competing one (see Fig. 2b for an example). This indicates that confirmation bias has an important role in our model, facilitating polarization of opinions, since agents are influenced little by peers with conflicting opinions. The effect is larger as λ increases, i.e. as under-reaction is growing, confirmation bias becomes more effective in generating polarization. This seems to indicate that a society that changes opinion slowly is more susceptible to polarisation due to confirmation bias.

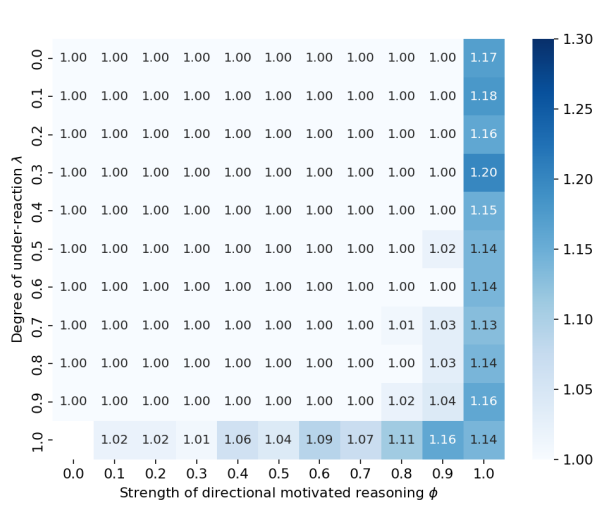
From the point of view of average opinions, overall values stay around the middle of the opinion interval (0.5), indicating that even if consensus is on one of the two models (optimistic or pessimistic), the exact model changes from one simulation to another, due to the symmetry of the model definition. That means that the belief distribution turns out to be characterised by path-dependence. This also applies in the case of two opinion clusters, when the opinion of the larger group changes from one simulation to another.

Another basic property of the model is that, while the model reaches consensus generally very fast, the time required to reach the stable state is increasing in λ (as shown in Fig. 3). In particular, as λ approaches 1, nodes take a larger number of iterations to reach a stable configuration, because each of them is less sensitive to new information and under-reacts. The relation between the convergence time and ϕ is, however, not so linear. For low λ values, large ϕ seems to slow down convergence, i.e. if the individuals do not under-react, then confirmation bias slows down convergence as society polarises. However, when λ is large, the opposite effect is visible: since individuals under-react, polarisation is faster as confirmation bias increases: they are not attracted by peers from the opposite opinion groups and so the simulations ends faster.

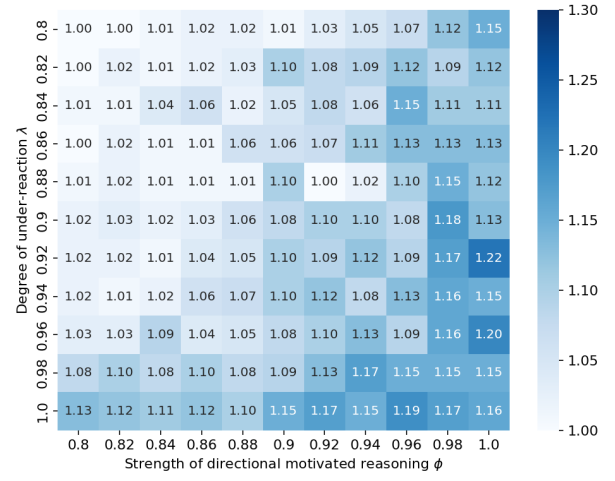
One-lobbyist scenario

In the second setting, we introduce a first lobbyist agent that randomly sends signals to the network, with the aim of shifting the distribution of subjective probabilities towards the model it is supporting (namely, towards the pessimistic model p).

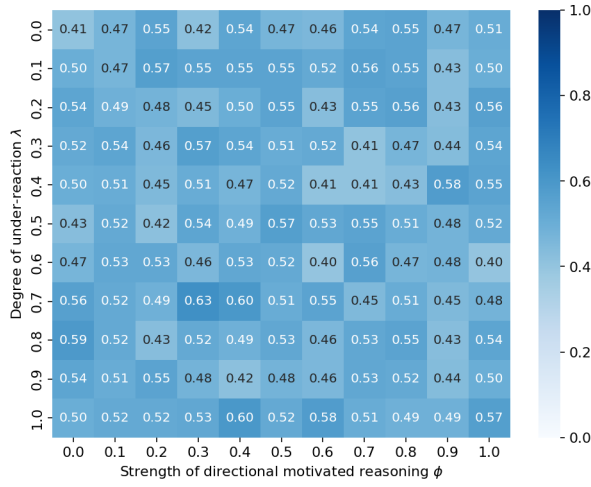
In Fig. 4 we show the number of clusters and average subjective probabilities at the steady state. While in the baseline simulation outcomes were balanced between the two competing models (thus resulting in average subjective probabilities around 0.5 across independent runs, see Fig. 1), in the one-lobbyist scenario average probabilities are significantly higher over the whole parameter space, indicating that generally the lobbyist is able to steer part of the network towards supporting its



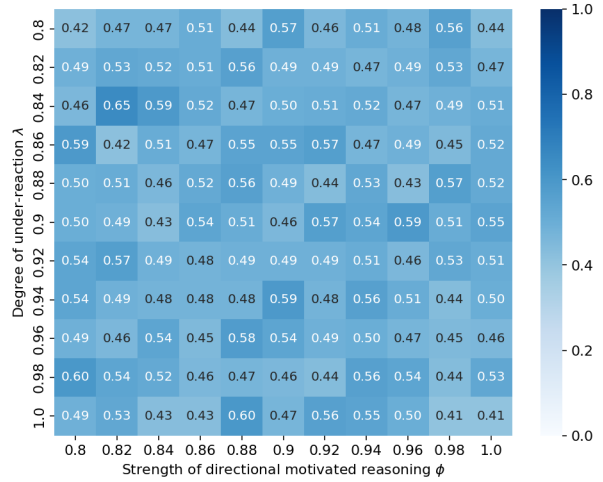
(a) Number of clusters: whole parameter space



(b) Number of clusters: zoom on the bottom right corner of the parameter space



(c) Average subjective probabilities: whole parameter space



(d) Average subjective probabilities: zoom on the bottom right corner of the parameter space

Figure 1. Baseline (no-lobbyist) scenario. The average effective number of clusters and the final average subjective probabilities of agents are represented as a function of strength of directional motivated reason ϕ and the degree of under-reaction λ . In the case where $\lambda = 1.0$, $\phi = 0.0$, the number of clusters metric is not applicable, since the agents do not change their opinions over time from their initial conditions, randomly drawn from a uniform distribution in $[0,1]$.

own preferred model. Furthermore, the effectiveness of the lobbyist is not globally homogeneous over the parameter space: in fact, the average probabilities are around 0.99 in the bottom-left corner region (indicating that, with these configurations, the lobbyist is successful in influencing the network almost always), but tend to be lower as we go farther off said region. Moreover, we see that the lobbyist, while managing to successfully increase the average subjective probability distributions, is unable to eliminate or reduce clustering in the bottom-right corner of the map (Fig. 4). If anything, in this region of the map an increase in polarization emerges, as the effective number of clusters increases, i.e. the size of the second cluster increases.

The described behaviour persists independently from the size N of the network, as reported in the Supplementary material where the results of the sensitivity analysis with respect to this parameter are analysed.

These results indicate that a society with medium-high levels of under-reaction and low confirmation bias (dark blue area in Fig. 4b) are easily influenced by a lobbyist, while when confirmation bias is strong or the population is more dynamic (low under-reaction), the peer-effect can contrast the lobbyist. This can generate consensus on the opposite opinion in a minority of simulations, when confirmation bias is low, or increase polarisation when confirmation bias is high. There appears to be a type of phase transition at the boundary, where the lobbyist does not have full power any more.

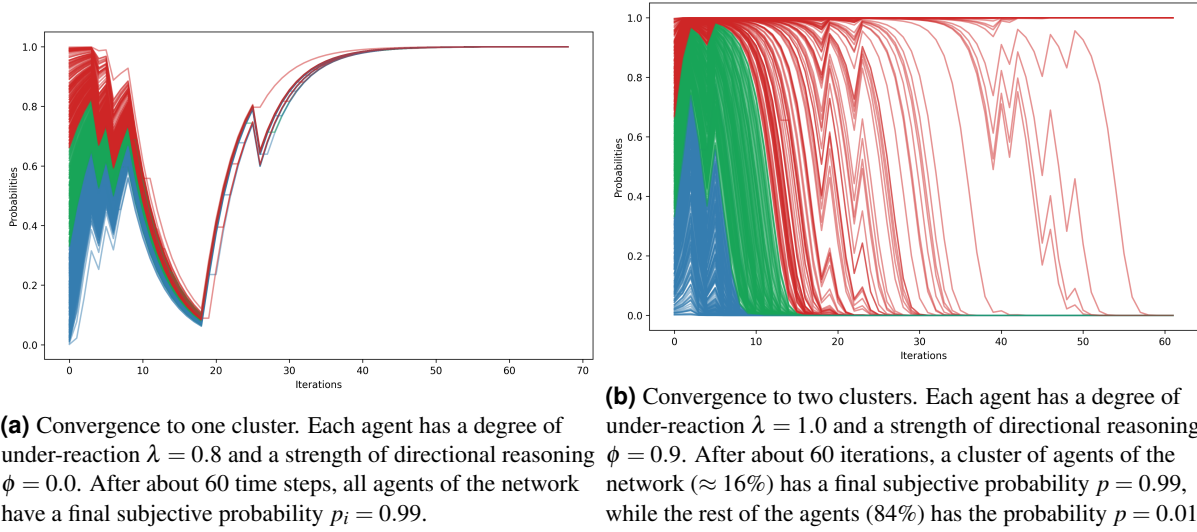


Figure 2. Opinion evolution. Examples of the evolution of the agent’s subjective probabilities in the baseline scenario with and without final consensus of the network.

To investigate further this phase transition, we performed budget sensitivity analysis, i.e. changed the budget of the lobbyist from 10,000 to 5,000, 20,000 and 40,000, allowing them to reach 10%, 40% and 80% of the population, respectively. In Fig. 5, we show the resulting average opinions. It is very clear that the region in which the lobbyist dominates the network (where it can impose its model almost certainly) expands linearly with the amount of resources available. With $B > 40,000$, the lobbyist influence is complete and global over the parameter space. Even outside said region, however, average subjective probabilities are higher on average the higher the budget allocated to the lobbyist, indicating that the lobbyist has still a non-negligible impact on the network behaviour regardless of the specific parameter configuration.

Two-lobbyists scenario

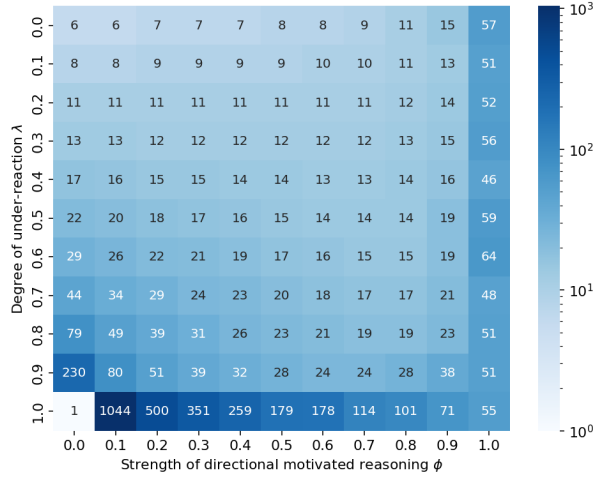
In the third scenario under consideration, we bring into the picture a second lobbyist which has the same endowments and the same kind of strategies as the first one, but that supports the competing model (namely, the “optimist” model).

In this configuration, average subjective probabilities across simulations seem to balance out, just as in the baseline scenario, as it is shown in Fig. 6b. In fact, as the two lobbyists are identical except for the specific model supported, their competing influence seems to cancel out, and the average subjective probabilities gravitates around $\bar{p} = 0.5$, regardless of the parametrization.

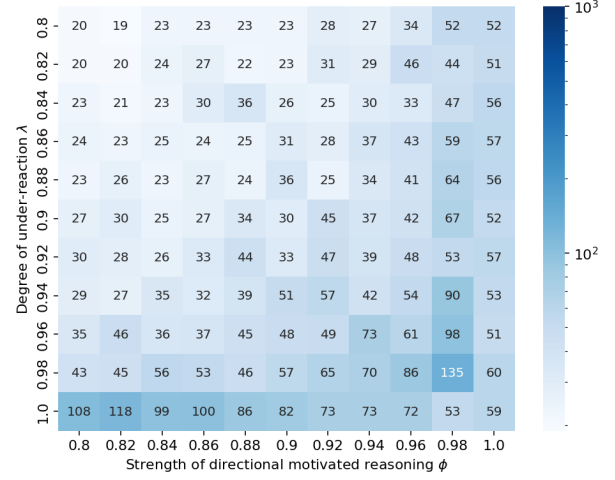
The main difference emerging in this case in comparison to the baseline scenario is that competition between lobbyists over influence on the network results in an increase in the number of iterations needed to reach the stable state, as shown in Fig. 6a.

In other words, the joint activity of the two lobbyists prevents the network from reaching an equilibrium as long as the lobbyists send signals; this is particularly true in the region that, in the previous scenario, was associated with higher lobbyist efficacy, strengthening the impression that, for that subset of configurations, it is easier for the external agent to project its influence on the network. Figure 6c represents the evolution of a typical run of the model in one of those configurations. Differently from what happens in the baseline scenario, where the network converges to a stable state relatively quickly, the system undergoes persistent fluctuations for as long as the two lobbyists are active on the network. Shortly after they stop emitting signals (beyond 100 iterations), however, the network quickly converges to a consensus, in a similar dynamics to what we have seen in the baseline scenario.

In this last scenario, we try to go a step further and we perform a sensitivity analysis on the lobbyists’ strategic horizon T with the aim of investigating how this parameter affects the model’s properties. Specifically, we run several different batches of simulations by increasing the parameter T to 1,000 and 2,000. At the same time, we adjust the lobbyists’ budget B accordingly, in order to maintain the same rate of signals per time-period of 20%, thus keeping the results comparable across specifications. As shown in Fig. 7, with a longer time horizon generally average opinion $\bar{p}$ remains about 0.5 in most configurations, suggesting that a longer horizon by the lobbyists does not radically change the outcome of the simulations. The average number of iterations, instead, tends to increase linearly with T in the bottom-left corner of the parameter space, the region where the lobbyists are most effective, while it does not seem to change much in the other regions, that converge very fast due to peer effects.

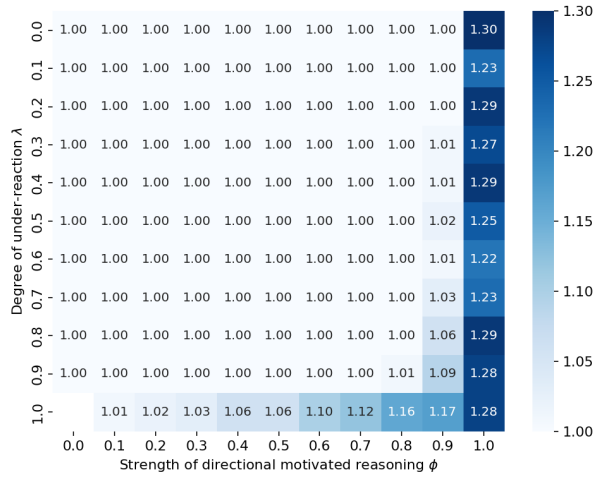


(a) Whole parameter space

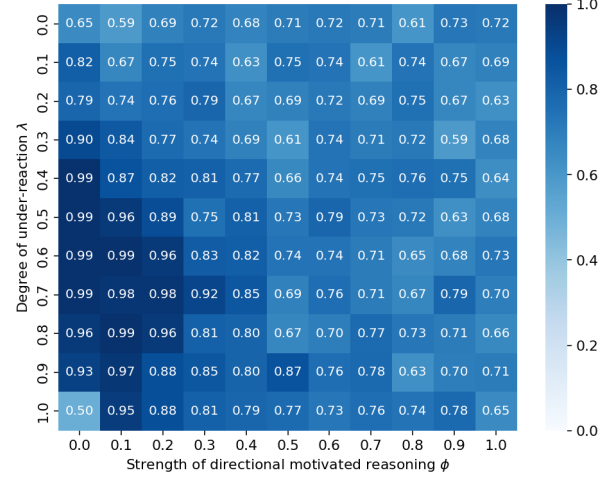


(b) Zoom on the bottom right corner of the parameter space

Figure 3. Average number of iterations in the baseline (no-lobbyist) scenario. The average number of iterations to reach out the equilibrium in the network is represented as a function of strength of directional motivated reason ϕ and the degree of under-reaction λ . The colorbar is in logarithmic scale.



(a) Average effective number of clusters.



(b) Average subjective probabilities.

Figure 4. One-lobbyist scenario. The effective number of clusters and average subjective probability of the final opinion distribution is represented as a function of the strength of directional motivated reason ϕ and the degree of under-reaction λ in presence of a lobbyist. The lobbyist supports the pessimistic model, has a budget $B = 10,000$ to send its signals and can be active for a time horizon $T = 100$.

An important result is that, despite the complete symmetry of our model and of the two lobbyist agents, when increasing T we observe the emergence of a locus of configurations in which average probabilities are markedly lower than 0.5 (see middle row panes of Fig. 7). This locus coincides with the transition between the two phases, where we switch from a prevailing lobbyist to a prevailing peer effect. This behaviour is more marked for longer T . In fact, for initial simulations with $T = 100$ it was not present, while it is clearly evident for $T = 1,000$, and $T = 2,000$. Therefore, it appears that, for very specific combinations of under-reaction in the range $[0.45, 0.85]$ and low confirmation bias, which sit at the border between the two regimes of the dynamics, the optimistic model prevails in the presence of two identical lobbyists. This suggests that the model has a mechanism that pushes the simulation towards a certain outcome in these specific situations: the long oscillatory behaviour as the lobbyists are active seems to drift towards the optimistic model, which will eventually prevail in a large fraction of simulations. In the supplementary material we give examples of the dynamics (see Figure S3) and we also show that this

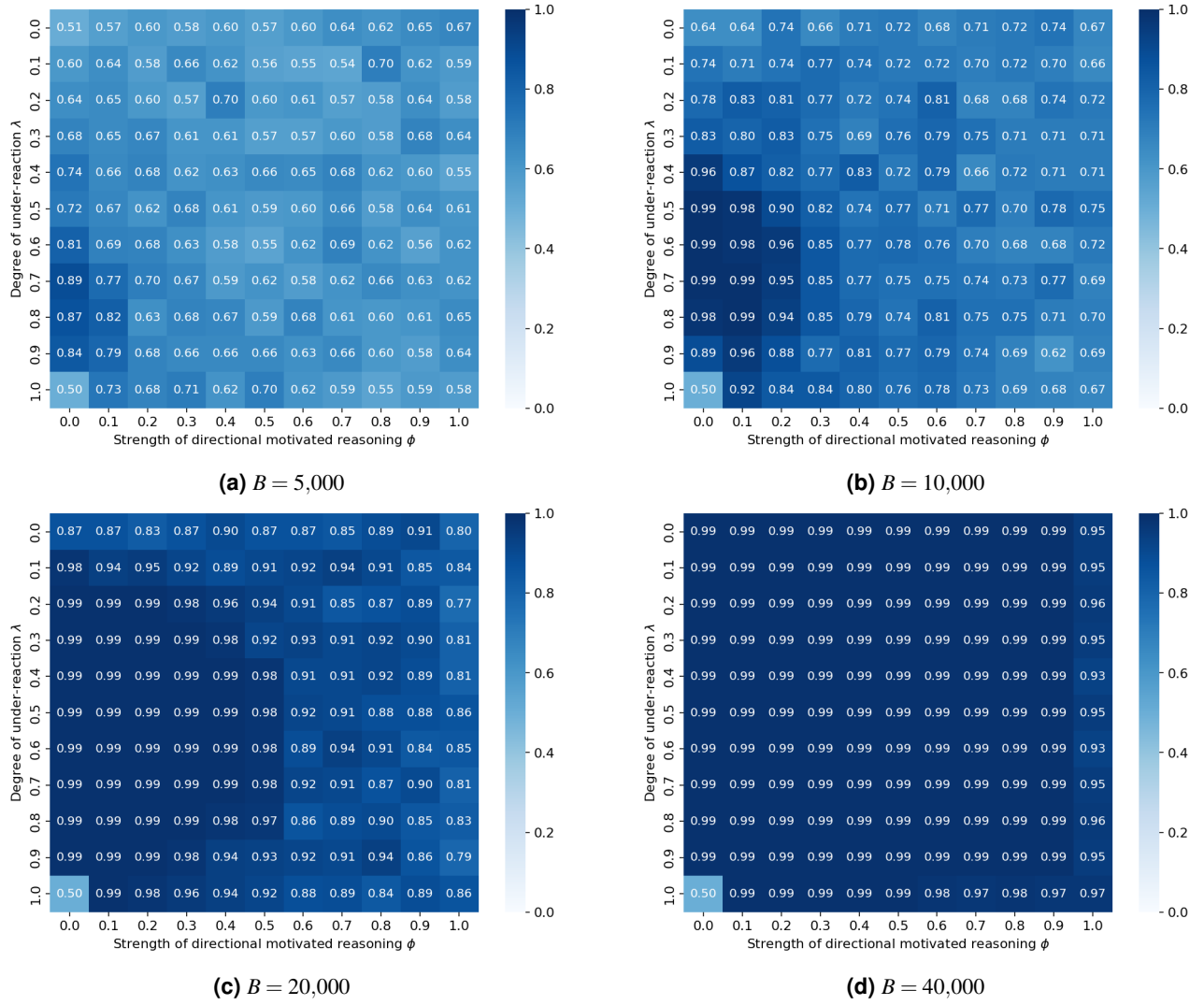
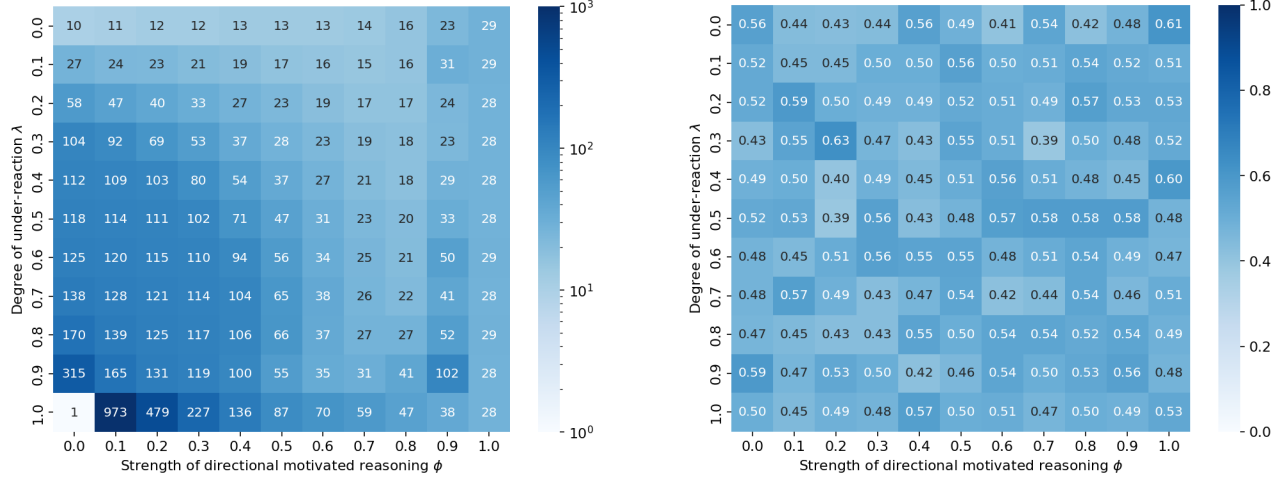


Figure 5. Budget sensitivity analysis in one-lobbyist scenario. The average subjective probability of the final opinion distribution is represented as a function of strength of directional motivated reason ϕ and the degree of under-reaction λ for different values of B , the budget of the lobbyist. The lobbyist supports the pessimistic model and can be active for a time horizon $T = 100$.

behaviour does not depend on the size of the network (Figures S4 and S5). In order to rule out numerical bias, we tried to invert the model specifications, which resulted in an inversion of the prevailing model in the same area. Therefore we cannot ascribe this effect to numerical loss of precision, but to the model dynamics. Further analyses are required to understand whether this behaviour can be associated with some real outcome in opinion formation.

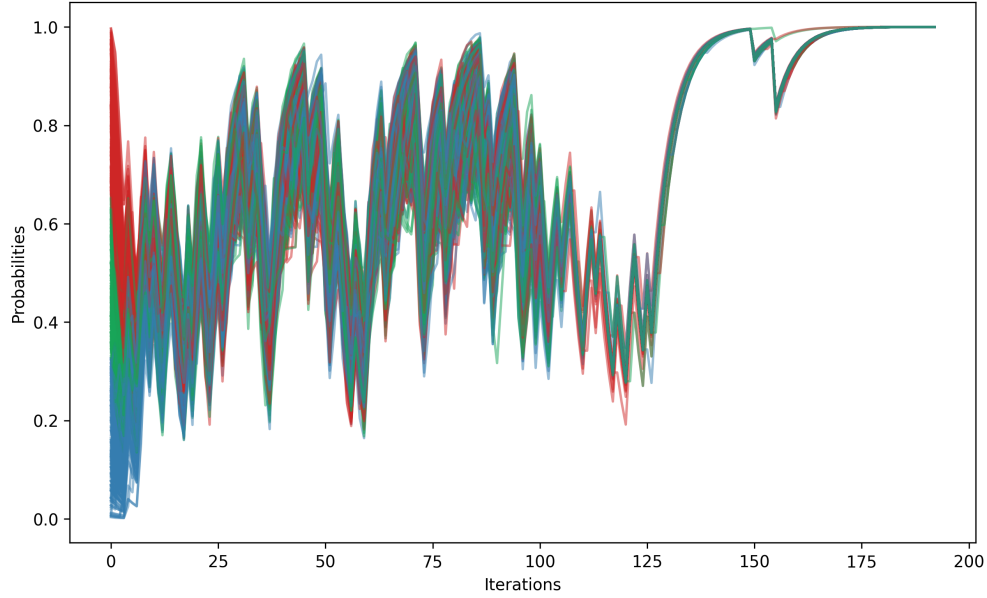
Discussion

In this work we introduced a novel model of opinion formation, where opinions change through a learning process that resembles Bayesian learning and is characterised by cognitive biases, specifically under-reaction and directional motivated reasoning (confirmation bias). The model includes lobbyists that are characterised by strategies (a set of agents to target at each iteration) and are budget constrained. After having provided some game theoretic examples, we studied the model numerically and observed the emergence of two polarised clusters when confirmation bias is large, even in the absence of lobbying. One lobbyist with an uniform strategy can attract the entire population to the opinion it promotes if the budget is large enough or if the individuals have low confirmation bias and medium under-reaction levels. Otherwise, a polarised minority continues to coexist with the cluster that agrees with the lobbyist. In the presence of two opposing lobbyists, the final steady states obtained are similar to those without lobbying, just that convergence happens only after the lobbyists stop influencing the network. As



(a) Average number of iterations.

(b) Average subjective probabilities.

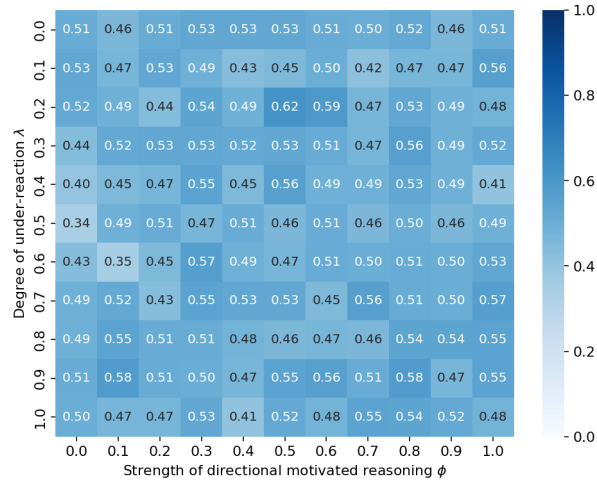


(c) Example evolution plot. Each agent has a degree of under-reaction $\lambda = 1.0$ and a strength of directional reasoning $\phi = 0.9$. Within the active time horizon (100 iterations), none of the lobbyists is able to definitely attract all agents opinions near it supporting models, thus the agents' opinions strongly oscillate. After that, the simulation evolves without any lobbyist action and the network quickly reaches a final consensus, as in the baseline scenario.

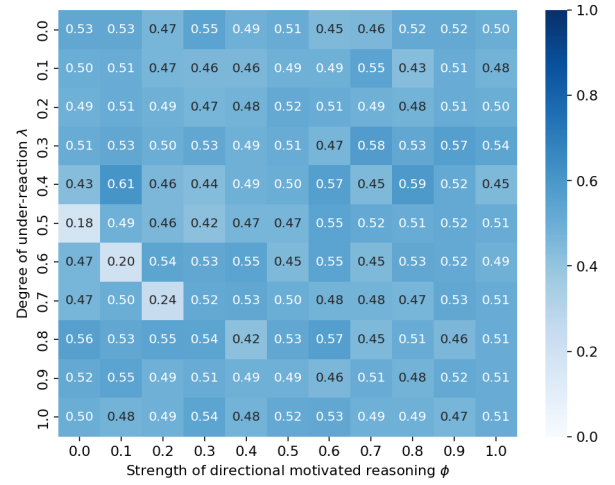
Figure 6. The two-lobbyists scenario. The average number of iterations and average subjective probabilities of the final opinion distribution are represented as a function of strength of directional motivated reason ϕ and the degree of under-reaction λ . The lobbyists supports opposing models (one the optimistic model and the other the pessimistic one). Both of them are endowed with a budget $B = 10,000$ to send its signals to the agents, i.e. can reach out an average of 20% of the agents in each time steps and can be active for a time horizon $T = 100$.

long as they are active, the population continues to oscillate between different opinions without converging. This suggests that continuous lobbying can preclude consensus or convergence.

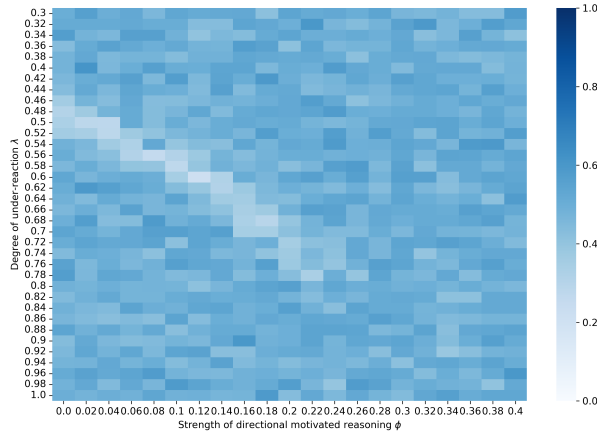
Our model shares some similarities with classical models of opinion dynamics, however exact mechanisms are different. The internal dynamics where an internal continuous weight is updated at discrete time steps is similar to bounded confidence models such as the Deffuant-Weisbuch³⁰ or Hegselman-Krause⁸ models. However, here, agents do not share the internal state with peers, but an external discrete opinion, more like in the CODA model³¹. Directional motivated reasoning, where agents react more strongly to opinions that are similar to their own, has a rationale similar to bounded confidence in the above



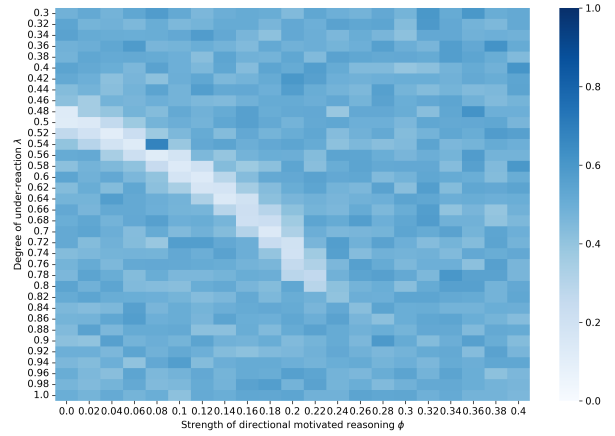
(a) Average subjective probabilities for $T = 1,000$



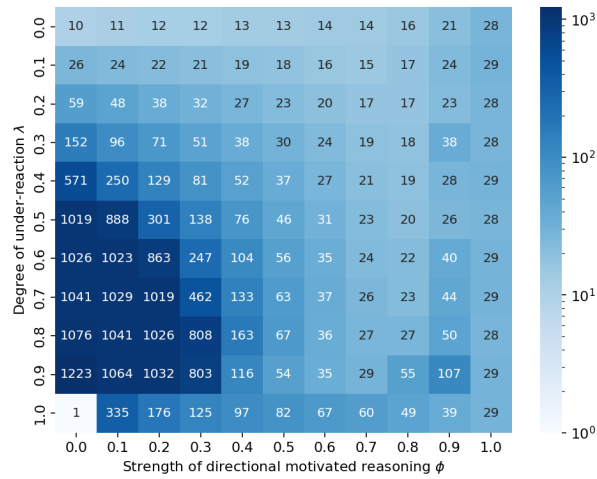
(b) Average subjective probabilities for $T = 2,000$



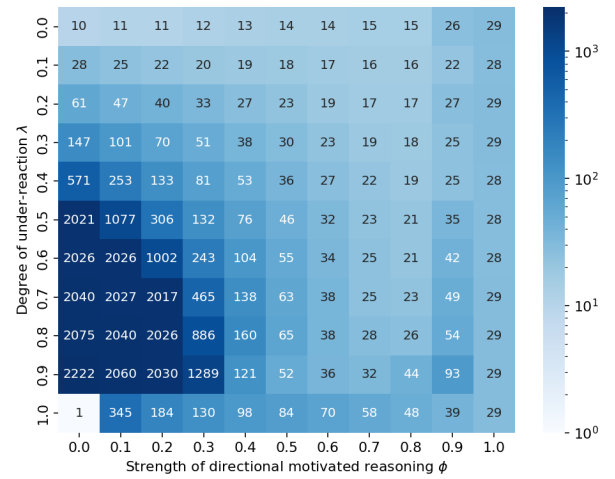
(c) Zoom of average subjective probabilities for $T = 1,000$



(d) Zoom of average subjective probabilities for $T = 2,000$



(e) Average number of iterations for $T = 1,000$



(f) Average number of iterations for $T = 2,000$

Figure 7. Long lobbyist time horizons. The average subjective probability of the final opinion distribution, and the average number of iterations are represented as a function of strength of directional motivated reasoning ϕ and the degree of under-reaction λ for different T , the maximum number of steps where lobbyists are active. The lobbyists support opposing models and have a fixed interaction rate with the agents of the networks for each step of the simulation, set at 0.2.

mentioned models, however in our model the effect is continuous rather than based on a discrete boundary. Indeed, a large ϕ leads to lack of consensus, similar to bounded confidence models, but we only obtain two clusters. Moreover, when consensus is reached, it usually stays on one of the two opinions, with all internal weights close to the range limit, not in the middle like bounded confidence models. This is more realistic, and models individuals who actually take a decision in the end, rather than remaining undecided and holding a moderate opinion ~ 0.5 . In this regard, our model is more similar to the voter model³² or to other discrete models.

Lobbyists are an important element of novelty in our model, since they are represented as more than an external field^{13–15,17,22} or zealot agent²³. Specifically, their effect on other agents can vary in time (at every iteration a subset of agents is reached) and is limited by a budget. This allows to model complex strategies and scenarios without changing the baseline model definition. We have shown that with this definition, one lobbyist attracts the complete population only in a certain area of the parameter space, a *lobbyist influence area*, where the population has low confirmation bias and medium levels of under-reaction. Polarisation appears as the population is more flexible (low under-reaction) or has more confirmation bias, where the *peer effect* can counteract the lobbyist influence. This polarising effect has also been seen in other models where extreme external information can cause minority extremist groups^{15,22}. A phase transition seems to appear at the boundary between these two regimes, i.e. lobbyist-influence area versus peer-effect area. When two symmetric opposing lobbyists are present, a rich behaviour is observed. During the active time horizon of the lobbyists, inside the lobbyist influence area, we observe fluctuations of opinions between the two extreme opinions, with convergence as soon as the lobbyists become silent. When oscillations are long, we observe an interesting convergence to the optimistic model for very specific parameter values at the boundary between the two regimes.

Future work will investigate more complex lobbyist strategies, extracted from real lobbying data³³. In particular, we are interested in their effect in the two different parameter regimes. Furthermore, the phase transition will be studied in more detail, to fully understand its dynamics and, ideally, find ways to validate it with real data. Further simulations are also planned to understand the dynamics on more realistic social networks, such as scale-free graphs that better represent real societies. Moreover, integration with statistical model checking environments such as MultiVeStA^{34,35} could provide further insight into model properties and dynamics.

Code availability

The presented mathematical model with behavioural bias was implemented in Python using the NDlib library³⁶ and is available in the GitHub repository: <https://github.com/ALMONDO-Project/ALMONDO-Model>. Simulations can be reproduced using the code in the same repository.

Data availability

We do not analyse or generate any datasets, because our work proceeds within a theoretical and mathematical approach. The data supporting the findings of this study are available within the paper and its supplementary information file.

References

1. Grossman, G. M. & Helpman, E. *Special interest politics* (MIT press, 2001).
2. Baumgartner, F. R., Berry, J. M., Hojnacki, M., Leech, B. L. & Kimball, D. C. *Lobbying and policy change: Who wins, who loses, and why* (University of Chicago Press, 2009).
3. Klüver, H. Informational lobbying in the european union: The effect of organisational characteristics. *West Eur. Polit.* **35**, 491–510 (2012).
4. Lewis, D. C. Advocacy and influence: lobbying and legislative outcomes in wisconsin. *Interest Groups & Advocacy* **2**, 206–226 (2013).
5. Schlichting, I. Strategic framing of climate change by industry actors: A meta-analysis. *Environ. Commun. A J. Nat. Cult.* **7**, 493–511 (2013).
6. Stokes, L. C. & Warshaw, C. Renewable energy policy design and framing influence public support in the united states. *Nat. Energy* **2**, 1–6 (2017).
7. Sobkowicz, P. Whither now, opinion modelers? *Front. Phys.* **8**, 587009 (2020).
8. Hegselmann, R., Krause, U. *et al.* Opinion dynamics and bounded confidence models, analysis, and simulation. *J. artificial societies social simulation* **5** (2002).
9. Centola, D. & Macy, M. Complex contagions and the weakness of long ties. *Am. journal Sociol.* **113**, 702–734 (2007).

10. Deffuant, G., Neau, D., Amblard, F. & Weisbuch, G. How can extremism prevail? a study based on the relative agreement interaction model. *J. Artif. Soc. Soc. Simul.* **5**, 1–8 (2002).
11. Watts, D. J. & Dodds, P. S. Influentials, networks, and public opinion formation. *J. consumer research* **34**, 441–458 (2007).
12. Bikhchandani, S., Hirshleifer, D. & Welch, I. Learning from the behavior of others: Conformity, fads, and informational cascades. *J. economic perspectives* **12**, 151–170 (1998).
13. Das, A., Gollapudi, S. & Munagala, K. Modeling opinion dynamics in social networks. In *Proceedings of the 7th ACM international conference on Web search and data mining*, 403–412 (2014).
14. Li, T. & Zhu, H. Effect of the media on the opinion dynamics in online social networks. *Phys. A: Stat. Mech. its Appl.* **551**, 124117 (2020).
15. Sîrbu, A., Loreto, V., Servedio, V. D. & Tria, F. Opinion dynamics: models, extensions and external effects. *Particip. sensing, opinions collective awareness* 363–401 (2017).
16. Sîrbu, A., Pedreschi, D., Giannotti, F. & Kertész, J. Algorithmic bias amplifies opinion fragmentation and polarization: A bounded confidence model. *PloS one* **14**, e0213246 (2019).
17. Pansanella, V., Sîrbu, A., Kertesz, J. & Rossetti, G. Mass media impact on opinion evolution in biased digital environments: a bounded confidence model. *Sci. Reports* **13**, 14600 (2023).
18. Fagiolo, G. & Roventini, A. Macroeconomic policy in dsge and agent-based models redux: New developments and challenges ahead. *J. Artif. Soc. Soc. Simul.* **20** (2017).
19. Haldane, A. G. & Turrell, A. E. Drawing on different disciplines: macroeconomic agent-based models. *J. Evol. Econ.* **29**, 39–66 (2019).
20. Bardoscia, M. *et al.* The impact of prudential regulation on the uk housing market and economy: Insights from an agent-based model. *J. Econ. Behav. & Organ.* **229**, 106839 (2025).
21. Roser-Renouf, C. & Maibach, E. W. Strategic communication research to illuminate and promote public engagement with climate change. *Chang. maintaining change* 167–218 (2018).
22. Sîrbu, A., Loreto, V., Servedio, V. D. & Tria, F. Opinion dynamics with disagreement and modulated information. *J. Stat. Phys.* **151**, 218–237 (2013).
23. Mobilia, M. Commitment versus persuasion in the three-party constrained voter model. *J. Stat. Phys.* **151**, 69–91 (2013).
24. Epstein, L. G., Noor, J. & Sandroni, A. Non-bayesian learning. *The BE J. Theor. Econ.* **10** (2010).
25. Massari, F. Under-reaction: Irrational behavior or robust response to model misspecification? Available at SSRN 3636136 (2020).
26. Bottazzi, G., Giachini, D. & Ottaviani, M. Market selection and learning under model misspecification. *J. Econ. Dyn. Control.* **156**, 104739 (2023).
27. Druckman, J. N. & McGrath, M. C. The evidence for motivated reasoning in climate change preference formation. *Nat. Clim. Chang.* **9**, 111–119 (2019).
28. Acemoglu, D. & Ozdaglar, A. Opinion dynamics and learning in social networks. *Dyn. Games Appl.* **1**, 3–49 (2011).
29. Nash, J. F. Equilibrium points in n -person games. *Proc. Natl. Acad. Sci.* **36**, 48–49, DOI: [10.1073/pnas.36.1.48](https://www.pnas.org/doi/pdf/10.1073/pnas.36.1.48) (1950). <https://www.pnas.org/doi/pdf/10.1073/pnas.36.1.48>.
30. Deffuant, G., Neau, D., Amblard, F. & Weisbuch, G. Mixing beliefs among interacting agents. *Adv. Complex Syst.* **3**, 87–98 (2000).
31. Martins, A. C. Continuous opinions and discrete actions in opinion dynamics problems. *Int. J. Mod. Phys. C* **19**, 617–624 (2008).
32. Clifford, P. & Sudbury, A. A model for spatial conflict. *Biometrika* **60**, 581–588 (1973).
33. Errichiello, G., Falcone, P. M. & Popoyan, L. Navigating climate policy: The influence of lobbying trends and narratives in europe. *Environ. Sci. & Policy* **163**, 103974 (2025).
34. Vandin, A., Giachini, D., Lamperti, F. & Chiaromonte, F. Automated and distributed statistical analysis of economic agent-based models. *J. Econ. Dyn. Control.* **143**, 104458 (2022).
35. Sebastio, S., Vandin, A. *et al.* Multivesta: Statistical model checking for discrete event simulators. In *7th International Conference on Performance Evaluation Methodologies and Tools, VALUETOOLS 2013; Torino; Italy; 10 December 2013 through 12 December 2013*, 310–315 (ACM, 2013).

36. Rossetti, G. *et al.* Ndlb: a python library to model and analyze diffusion processes over complex networks. *Int. J. Data Sci. Anal.* **5**, 61–79 (2018).

Acknowledgements

This study received funding from the European Union - Next-GenerationEU - National Recovery and Resilience Plan (NRRP) – MISSION 4 COMPONENT 2, INVESTMENT N. 1.1, CALL PRIN 2022 PNRR D.D. 1409 14-09-2022 – ALMONDO Project, CUP N. J53D23015400001.

Author contributions statement

D.G. conceived the mathematical model and V.P. implemented it in NDlib Python library. All authors conceived the experiments to explore its properties. V.D.R and L.C. performed the simulations. All authors analysed and interpreted the results. D.G., L.P., L.C. and A.S. wrote the first draft of the manuscript. All authors reviewed and approved the manuscript.

Additional information

Supplementary Information The online version will contain supplementary material

Competing interests The authors declare no competing interests.